Con. 7885-13. **GX-12071**

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question no. **1** is **compulsory**.
(2) Attempt any **three** questions out of the remaining **five** questions.
(3) **Figures** to **right** indicate **Full** marks.

1. (a) Prove that real and imaginary parts of an analytic function F(z) = u + iv are harmonic function. **5**

(b) Find fourier series for $f(x) = |\sin x|$ in $(-\Pi, \Pi)$. **5**

(c) Find the Laplace transform of $\int_0^t ue^{-3u} \sin 4u du$ **5**

(d) If $\bar{F} = xye^{2z}\,\hat{i} + xy^2\cos z\,\hat{j} + x^2\cos xy\,\hat{k}$, find div $\bar{F}$ and curl $\bar{F}$. **5**

2. (a) Using Laplace transform, solve :– **6**
$(\theta^2 + 3\theta + 2)y = e^{-2t}\sin t$ where y(0) = 0, y′(0) = 0.

(b) Find the directional derivative of $d = x^2y\cos z$ at $(1, 2, \Pi/2)$ in the direction of $\bar{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ **6**

(c) Find the fouries series expansion for $F(x) = \sqrt{1 - \cos x}$ in $(0, 2\Pi)$, Hence deduce that $\frac{1}{2} = \sum \frac{1}{4^{n2} - 1}$. **8**

3. (a) Prove the $J_{3/2}(x) = \sqrt{\frac{2}{\Pi x}}\left\{\frac{\sin x}{x} - \cos x\right\}$. **6**

(b) Evaluate by green's theorem, $\oint_C (x^2y dx + y^3 dy)$ Where C is the closed path formed by y = x, y = x². **6**

(c) (i) Find Laplace transform of $\frac{\cos bt - \cos at}{t}$ **4**

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(ii) Find Laplace transform of :– $\frac{d}{dt}\left[\frac{\sin t}{t}\right]$ **4**

4. (a) Show that the set of functions $\{\sin x, \sin 3x.......\}$ OR $\{\sin(2n+1)x : n = 0, 1, 2, ...\}$ is orthogonal over $\left[0, \Pi/2\right]$, Hence construct orthonormal set of functions. **6**

(b) Find the imaginary part whose real part is $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ **6**

(c) Find inverse Laplace transform of :– **8**

(i) $\log\left(\frac{s^2 + 4}{s^2 + 9}\right)$

(ii) $\frac{s}{(s^2+4)(s^2+9)}$

5. (a) Obtain half range sine series for $f(x) = x^2$ in $0 < x < 3$. **6**

(b) A vector field $\bar{F}$ is given by $\bar{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ is irrotational and Hence find scalar point function ϕ such that $\bar{F} = \nabla\phi$ **6**

(c) Show that the function $V = e^x(x\sin y + y\cos y)$ satisfies Laplace equation and find its corresponding analytic function and its harmonic conjugate. **8**

6. (a) By using stoke's theorem, evaluate $\oint_C \left[(x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j}\right].d\bar{r}$ where 'C' is the boundary of the region enclosed by circles $x^2 + y^2 = 4$, $x^2 + y^2 = 16$. **6**

(b) Show that under the transformation $w = \frac{s - 4z}{4z - 2}$ the circle $|z| = 1$ in the z–plane is transformed into a circle of unity in the w–plane. **6**

(c) Prove that $\int J_3(x)\,dx = \frac{-2J_1(x)}{x} - J_2(x)$. **8**
