P4-RT-Exam.-Feb.-13-2-243

Con. 6389-13. **GS-6075**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from remaining.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find Laplace Transform of $t\sqrt{1+\sin 2t}$. **5**

(b) Determine nature of poles and find residue at each pole of – **5**

(i) $f(z) = z^2 \; e^{1/z}$

(ii) $g(z) = \dfrac{1-e^{2z}}{z^3}$

(c) Obtain complex from of Fourier series for $f(x) = e^{-x}$ in $(-1, 1)$. **5**

(d) Determine constants a, b, c, d, e if $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i\,(4x^3y - exy^3 + 4xy)$ is analytic. **5**

2. (a) Using Laplace transforms evaluate **6**

$$\int_0^{\infty} e^{-t} \left[\int_0^t \frac{\sin u}{u} \, du \right] dt$$

(b) Show that $u = x^4 - 6x^2y^2 + y^4$ is harmonic. Hence constant analytic function $f(z) = u + iv$ and write harmonic conjugate of u. **7**

(c) Find Fourier series expansion for – **7**

$$f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12} \text{ in } (0, 2\pi).$$

Hence deduce that $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \ldots\ldots$

[TURN OVER

3. (a) Evaluate $\int_c \frac{3z^2 + z}{z^2 - 1} dz$ where 'C' is the circle $|z| = 2$. **6**

(b) Find inverse Laplace Transform of – **7**

(i) $\frac{3 + 29}{(s+4)(s^2+9)}$ (ii) $\log\left[\frac{s+a}{s+b}\right]$

(c) Find half range Cosine series for $f(x) = x$, $0 < x < 2$. Hence deduce that – **7**

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots$$

4. (a) State true or false with proper justification. There does not exist an analytic function whose real part is $(x^3 - 3x^2y - y^3)$. **6**

(b) Find fourier series expansion for – **6**

$$f(x) = \begin{cases} 0 & -2 < x < -1 \\ 1+x & -1 < x < 0 \\ 1-x & 0 < x < 1 \\ 0 & 1 < x < 2 \end{cases}$$

(c) Find Laplace transform of – **8**

(i) $\frac{e^{-at} - \cos at}{t}$ (ii) $t^{-1}\int_0^t e^{-u} \sin\theta \, du.$

5. (a) If $f(a) = \int_c \frac{4z^2 + z + 5}{z - a} dz$ where C is $|z| = 2$ **6**

Find $f(1)$, $f(i)$, $f'(-1)$, $f''(-i)$

(b) Find the bilinear transformation which maps the points 1, i, –1 onto the points i, 0, –i. **6**

(c) Find Fourier cosine integral for – **8**

$$f(x) = \begin{cases} 1 - x^2 & 0 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

Hence evaluate – $\int_0^\infty \frac{\sin\omega - \omega\cos\omega}{\omega^3} \cos\left(\frac{\omega}{2}\right) d\omega$

6. (a) Using convolutions theorem find inverse Laplace transform of $\frac{s}{(s^2+4)^2}$. **6**

(b) Find the map of real axis in z-plane under the transformation $w = \frac{1}{z+i}$. **6**

(c) Obtain Laurent's Series expansion for $f(z) = \frac{1}{z(z+1)(z-2)}$ about z = 0 in the region. **8**

(i) $1 < |z| < 2$ (ii) $|z| > 2$

7. (a) If $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$ **6**

Prove that Cauchy Riemann equations are satisfied at z = 0 but f(z) is not analytic at z = 0.

(b) Using residue theorem - evaluate **8**

(i) $\int_0^{2\pi} \frac{\cos 3\theta \, d\theta}{5 - 4\cos\theta}$

(ii) $\int_{-\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$

(c) Using Laplace transforms solve – **6**

$$\frac{d^2y}{dt^2} + 9y = 18t, \; y(0) = 0, \; y\left(\frac{\pi}{2}\right) = 0$$
