Con. 6295-13.   **LJ-10451**

(3 Hours)   [**Total Marks : 100**

N. B. : (1) **Question No. 1** is **compulsory**.
      (2) Attempt any **four** questions from the remaining **six** question.
      (3) Assume suitable data if necessary & justify the same.
      (4) Statistical table will be provided, if required.

1. (a) Prove that the eigen values of on ozthogonal matrix are $+1$ or -1.
    (b) If $\bar{F}=(x+zy+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+6y+2z)\hat{K}$ is solenoidal, find a, b and c.   **5**
    (c) Show that the matrix   **5**

$$A=\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix} \text{ is}$$ **5**

   non-derogatory.
    (d) Let x be a continuous random variable with p.d.f. $f(x) = k(x-x^2)$ $0<x<1$, findk, mean,

5

2. (a) Find mean and variance of binomial distribution
    (b) Reduce the following matrix to normal form and hence find its rank.   **6**

$$A=\begin{bmatrix} 1 & 2 & 3 & 7 \\ 4 & 2 & -1 & 2 \\ 3 & 2 & 1 & 2 \\ 4 & 2 & 1 & 3 \end{bmatrix}$$ **6**

    (c) Prove that $\bar{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3z^2\hat{K}$ is a conservative field, find its scalar potential
        and also work-done in moving particle from $(1, -2, 1)$ to $(3, 1, 4)$ **8**

3. (a) Test the consitency and if consistent, solve, the following system of equations.   **6**
         $2x-3y+7z =5$
         $3x+y-3z =13$
         $2x-19y+47z =32$

    (b) Using Green's theorem evaluate $\oint_c (e^{x^2} - xy)dx - (y^2 - ax)dy$ where C is the circle
        $x^2+y^2 = a^2$ **6**

    (c) The marks of 1000 students of a university are found to be normally distributed
        with mean 70 and standard deviation 5. Estimate the number of students whose marks   **8**
        will be (i) between 60 and 75, (ii) more than 75.

4. (a) Find eigen values and eigen vectors of A^3, where   **6**

$$A=\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

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(b) By using stokes theorem, evaluate   **6**

$$\oint_c [(x^2 + y^2)i + (x^2 - y^2)j].dr$$

Where C is the boundary of the region enclosed by circles $x^2+y^2=4$ & $x^2+y^2=16$.   **8**

(c) Calculate coefficient of correlation between x and y where

X	2	4	5	6	8	11
y	18	12	10	8	7	5

5. (a) Verify Caylay-Hamilton's theom. For the matrix A, hence find A^{-1} if exist and A^4.   **6**

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 1 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

(b) If $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$ is orthogonal, find a, b, c   **6**

(c) Reduce the following quadratic form to canonical form by congruent transformation State its rank signature index.   **8**

$$x^2 - 2y^2 + 3z^2 - 4yz + 6zx$$

6. (a) Show that the matrix $A = \begin{bmatrix} 1 & -6 & 4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ is   **6**

diagonalizable. Also find its transforming matrix and the diagonal matrix.

(b) Find Spearman's rank coefficient of correlation for the following data.   **6**

X	1	3	4	5	7	8	10
y	20	16	14	10	6	6	2

(c) Using Gauss Divergence theom, evaluate   **8**

$$\iint_s [4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}].d\bar{s}$$ where s is the region bounded by $y^2=4x$, x=1, z=0, z=3.

7. (a) A random variable x has probability density function ½x x = 1, 2, 3, ... Find its mean and variance   **6**

(b) If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ find A^{100} and e^A.   **6**

(c) Obtain the equation of the line of regression of y on x and the line of regression of x on y from the following data.   **8**

X	70	72	74	76	78	80
y	163	170	179	188	196	220