Con. 8032-13. **LJ-10463**

(3 Hours) **[Total Marks : 100**

1. Attempt any **five** questions :-
   (a) Show that there does not exist any analytic function. f(z) = u+iv such that $u+v = \frac{x-y}{x+y}$ **5**

   (b) Find the poles of $f(z) = \frac{\sec z}{z^2}$ which lie inside the circle $C: |z|=2$. **5**
   Also find the residues of f(z) at these poles.

   (c) Show that $\frac{d}{dx}\left[x^{\frac{n}{2}} J_n(\sqrt{x})\right] = \frac{1}{2} x^{\frac{n-1}{2}} J_{n-1}(\sqrt{x})$ **5**

   (d) A is a 3 x 3 matrix whose characteristic polymial is $\lambda^3 + 2\lambda^2 + 3\lambda + 4$. Find the sum of the eigen values of A^{-1}. **5**

2. (a) Show that the bilinear transformation **6**
   $w = \frac{9z+3i}{3-iz}$ maps $|z| \leq |\text{onto}| w| \leq 3$

   (b) Show that the matrix is not digonalisable. **6**

$$A = \begin{bmatrix} -17 & 18 & -6 \\ -18 & 19 & -6 \\ -9 & 9 & 2 \end{bmatrix}$$

   (c) Show that $\vec{F} = \frac{\vec{r}}{r^3}$ is irrotational i also find the corresponding potential function. **8**

3. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using the residue theorem. **6**

   (b) If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ show that $e^{At} = \begin{bmatrix} \cos t & -\sin t \\ \sin t & \cos t \end{bmatrix}$ **6**

   (c) Verify Green's therem for **6**
   $\int_C (x2-y^2)\,dx + (x^3+y^3)\,dy$
   over the region bounded by $1 \leq x \leq 2$ and $1 \leq y \leq 3$

4. (a) Show that $J_2(X) = J_0''(X) - \frac{1}{X} J_0'(X)$ **6**

   (b) Evaluate $\int_C (x^2+2y)dx + (4x+y^2)\,dy$ over the region bounded by $y=0$, $y=2x$, $x+y=3$ **6**

(c) $A = \begin{bmatrix} 2 & a & b \\ 0 & 2 & c \\ 0 & 0 & 3 \end{bmatrix}$ show that A is diagonalisabel if any only if A is derogatory. **8**

5. (a) $A = \frac{1}{5} = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$ **6**

Show that the eigentralnes are of unit modulus and the eigenvectors are orthogonal.

(b) Find a and b such that $u = (5x + 3y)(2x^2 + axy + by^2)$ is a harmanic function. **6**

(c) Find the analytic function f (z) whose real part is $u = \dfrac{2\sin x.\cosh y}{\cosh 2y - = \cos 2x}$ **8**

6. (a) Evaluate $\int_C \overline{Z}$ dz over the upper half of C : $|z| = 2$, traversed in the anti-clockwise direction. **6**

(b) Verify the Gauss divergence theorem for $\vec{F} = (x^2 - yz) + (y^2 - zx)\uparrow + (z^2 - xy)\uparrow$ over the surface S : $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$ **6**

(c) Find the laurent series expansion of $f(z) = \dfrac{1}{(Z+1)(Z+3)}$ in **8**

(i) $|z| < |,$

(ii) $|z| > 3,$

(iii) $0 < |z+1| < 2$

7. (a) Verify Stocks theorem for $\vec{F} = y\hat{i} + z\hat{f} + x\hat{K}$ where S is the upper hemisphere $x^2+y^2+z^2 = 1,\ 2>0.$ **6**

(b) Diagonalise the quadratic form $Q = 2xy+2xz - 2yz$ using an arthogonal transformation. **6**

(c) Show that **8**

$$\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x)\right] = 2\left[\frac{n}{x} J_n^2(x) - \frac{(n+1)}{x} J_{n+1}^2(x)\right]$$