48 : 1st half.13-AM(v)

Con. 6430-13. **GS-6900**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Show that $J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left\{\frac{3-x^2}{x^2}\sin(x) - \frac{3}{x}\cos(x)\right\}$ **5**

(b) Show that matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is non-derogatory. **5**

(c) Evaluate $\oint_c \frac{1}{\left(z^3-1\right)^2}dz$ where 'c' is $|z-1| = 1$ **5**

(d) Evaluate $\int_A^B \left(3x^2y-2xy\right)dx+\left(x^3-x^2\right)dy$ along $y^2 = 2x^3$ from A(0, 0) and B(2, 4) **5**

2. (a) Prove that $xJ_n^1(x) = -nJ_n(x) + xJ_{n-1}(x)$ **6**

(b) Show that the matrix $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ is diagonalizable. Also find the transforming matrix and diagonal matrix. **7**

(c) Evaluate $\int\int_c (\nabla \times \overline{F})\cdot d\overline{s}$ where **6**

$\overline{F} = (2x-y+z)i+\left(x+y-z^2\right)j+(3x-2y+4z)k$ and 's' is the surface of the cylinder $x^2 + y^2 = 4$ bounded by the plane $z = 9$ and open at the other end.

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3. (a) Evaluate $\int_c \frac{z+1}{z^3-2z^2}dz$ where 'c' is 7

   (i) the circle $|z-2-i| = 2$

   (ii) the circle $|z-1-2i| = 2$

   (b) Show that $\overline{F} = \left(ye^{xy}\cos(z)\right)i + \left(xe^{xy}\cos(z)\right)j - \left(e^{xy}\sin(z)\right)k$ is irrotational and find the scalar potential for $\overline{F}$ and evaluate $\int \overline{F}\cdot d\overline{r}$ along the curve joining the points (0, 0, 0) and (–1, 2, π) 7

   (c) prove that 6

$$\int J_3(x)dx = \frac{-2J_1(x)}{x} - J_2(x)$$

4. (a) Define Analytic function. State and prove Cauely-Riemann equation in polar co-ordinates. 7

   (b) Verify Gauss-Divergence Theorem. Evaluate for $\overline{F} = (2x)i + (xy)j + z(k)$ over the region bounded by the cylinder $x^2 + y^2 = 4$, $t = 0$, $t = 6$ 7

   (c) If $A = \begin{bmatrix} 1 & 2 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ find A^{100} 6

5. (a) Define conformal mapping. Find Bilinear transformation which maps the prints z = 0, i, –1 onto w = i, 1, 0. 7

   (b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{x^6+1}dx$ 7

   (c) If $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. Find the characteristic roofs and characteristic vectors of $A^3 + I$ 6

6. (a) Find all possible Laurent's series expansion of the function $f(z) = \dfrac{1}{z^2(z-1)(z+2)}$ 7

about $z = 0$ for (i) $|z|<1$, (ii) $1<|z|<2$, (iii) $|z|>2$

(b) If $f(z) = u + iv$ is analytic and $u+v = \dfrac{2\sin(2x)}{e^{2y}+e^{-2y}-2\cos(2x)}$ find $f(z)$. 7

(c) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and hence find the matrix 6

$2A^5 - 3A^4 + A^2 - 4I$.

7. (a) Prove that the circle $|z| = 1$ in the z-plane is mapped onto the cordinate in the w-plane under the transformation $w = z^2 + 2z$. 7

(b) Reduce the following quadratic form to Canonical form and find its rank and signature 7

$$x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_1x_3 + 2x_2x_3$$

(c) Verify Green's Theorem for 6

$\int_c \left(\frac{1}{y}dx + \frac{1}{x}dy\right)$ where 'c' is the boundary of the region defined by

$x = 1, x = 4, y = 1$ and $y = \sqrt{x}$.
