(3 Hours)

[Total Marks : 100

N.B. (1) Question No. **1** is **compulsory**.

(2) Attempt any **four** questions out of remaining **six** questions.

(3) Assume any **suitable** data when **required** but justify the **same**.

1. (a) Define a strict – sense stationary (SSS) and wide-sense stationary (WSS) Random Process.   5

  (b) Show that the conditional probability satisfies the axioms of probability.   5

  (c) State and explain Total probability theorem and Baye's Theorem.   5

  (d) State Central limit theorem and give it's significance.   5

2. (a) What is CDF of a Random variable ? State and prove the properties of Distribution functions.   10

  (b) It is known that the screws produced by a certain company will be defective with probability 0·01 independently to each other. The company sells the screws in the packages of 10 and offers a money-back guarantee that at most 1 of the 10 screws is defective. What proportion of packages sold must company replace ?   5

  (c) The probability of hitting an aircraft is 0·001 for each shot. How many shots should be fired so that the probability of hitting with two or more shots is above 0·95 ?   5

3. (a) Define characteristic function of a Random variable. Prove that given the characteristics function the nth moment is given by   10

$$E\left[x^n\right] = \frac{1}{j^n} \frac{d^n}{dw^n} \phi_x \left(w\right)\Big/_{w = 0}.$$

  (b) Suppose pdf of x, $f_x(x) = \frac{2x}{\pi^2}$, $0 < x < \pi$, and $Y = \sin x$. Determine the PDF of Y.   10

4. (a) Find the normalization constant C and the marginal pdf's for the following joint pdf -   10

$$f_{xy}(x, y) = \begin{cases} c\, e^{-x}\, e^{-y} & 0 \le y \le x < \infty \\ 0 & \text{elsewhere} \end{cases}.$$

  (b) Explain in brief :—

    (i) Poisson process

    (ii) Gaussian process.   10

[**TURN OVER**

5. (a) Explain what is Random process. Define Ensemble mean, Auto correlation and Auto **10**
        covaraiance of the process in terms of Indexed Random variables in usual
        Mathematical forms.

   (b) Consider the Random phase Sinusoid given by $X(t) = A \cos (wt + \phi)$ where A and w **10**
        are constants and $\phi \sim u$ $[0 \cdot 2\pi]$ is a Random variable. Prove that Random phase sinusoid
        is Ergodic in both mean and Auto correlation.

6. (a) Let $Z = X + Y$. Determine PDF of Z $f_z(z)$.   **10**

   (b) A stationary process is given by $x(t) = 10 \cos [100t + \theta)$ where θ is a Random **10**
        variable with uniform probability distribution in the Interval $[-\pi, \pi]$. Show that it is
        wide-sense stationary [WSS] process.

7. (a) State and prove the Chapman-Kolmogorov equation.   **10**

   (b) The transition probability matrix of Markov chain is –   **10**

$$
\begin{array}{c c c c}
 & 1 & 2 & 3 \\
\begin{array}{c} 1 \\ 2 \\ 3 \end{array} &
\left[\begin{array}{c c c}
0 \cdot 5 & 0 \cdot 4 & 0 \cdot 1 \\
0 \cdot 3 & 0 \cdot 4 & 0 \cdot 3 \\
0 \cdot 2 & 0 \cdot 3 & 0 \cdot 5
\end{array} \right]
\end{array}
$$

   Find the limiting probabilities.

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