Con. 6481-13. **LJ-10243**

(3 Hours) **[Total Marks : 100**

N.B. : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions out of the remaining **six** questions.
(3) **Figures** to the **right** indicate **full** marks.

1. (a) Find the constant 'P' if $f(z) = r^2\cos(2\theta) + i\, r^2\sin(p\theta)$ is analytic. **5**
   (b) Find the image of the circle $|z| = 2$ un der the transfromation $w = z + 3 + 2i$. **5**
   (c) Find the laplace transform of $\cos^5(f)$. **5**
   (d) If A is nonsingular matrix of order 'n' then show that $adj \cdot adj(A) = |A|^{(n-2)} A$. **5**

2. (a) Find the eigen values and eigen vectors corresponding to the following matrix A where **6**

$$A = \begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$$

   (b) Find an analytic function f(z) whose imaginary part is $\dfrac{\sin(2x)}{\cosh(2y) + \cos(2x)}$. **6**
   (c) Find the laplace transfrom of the following :— **8**

   (i) $\dfrac{e^{-t}\sin(t)}{t}$ (ii) $\displaystyle\int_0^t u^2 \sin(u)\, du.$

3. (a) Test for consistency and solve :— **6**

$$5x_1 + 3x_2 + 7x_3 = 4$$
$$7x_1 + 2x_2 + 10x_3 = 5$$
$$3x_1 + 26x_2 + 2x_3 = 9$$

   (b) Find the bilinear transformation which maps the points $-1, 1, \infty$ onto the points $-1, -1, i$. **6**
   (c) Find the inverse laplace transform of the following :— **8**

   (i) $\log\left(\dfrac{s^2 + a^2}{\sqrt{s + b}}\right)$ (ii) $\dfrac{(s+1)\, e^{-s}}{(s^2 + s + 1)}$

4. (a) Find non-singular matrices P and Q such that $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ is reduced to normal form. Also find its rank. **6**

   (b) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)^3}$. **6**

   (c) (i) Find $L^{-1}\left\{\dfrac{3s + 1}{(s + 1)(s^2 + 2)}\right\}$ **4**

   (ii) Evaluate $\displaystyle\int_0^{\infty} e^{\left(-\sqrt{2}\right)t} \sin(t)\, \sinh(f)dt.$ **4**

[TURN OVER

5. (a) Using laplace transform solve the following $(P^2 + 2D + 5)\, y = e^{-t} \sin(t)$ when $y(0) = 0$ and $y'(0) = 1$. **6**

   (b) Find the orthogonal trajectory of the family of curves $e^{-x}(x \sin(y) - y \cos(y) = C$. **6**

   (c) (i) Evaluate $\int_C \frac{dz}{z^3 (z+4)}$ where 'C' $|z| = 2$. **4**

   (ii) Evaluate $\int_C \frac{z-1}{(z+1)^2 (z-2)}\, dz$ where 'C' encloses both poles of f(z). **4**

6. (a) Examine whether the vector $x_1 = (1, 2, 4)$ $x_2 = (2, -1, 3)$ and $x_3 = (0, 1, 2)$ are linearly independent. **6**

   (b) Find the image of the rectangular hyporbola $x^2 - y^2 = 1$ under the transformation $w = 1/z$. **6**

   (c) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the regions : **8**

   (i) $1 < |z - 1| < 1$ (ii) $1 < |z - 3| < 2$ (iii) $|z| < 1$

7. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 5\cos(\theta)}$. **6**

   (b) If $A = \begin{bmatrix} 2 & 3 \\ -3 & -4 \end{bmatrix}$ Find A^{50}. **6**

   (c) Verify Cayley-Hamilton theorem for the matrix A and there find A^{-1} and A^4 where : **8**

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

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