

(3 Hours)

[ Total Marks : 100

**N.B.** (1) Question No. 1 is **compulsory**.(2) Attempt any **four** questions from the remaining **six** questions.

1. (a) Determine whether the function  $f(z) = \cosh z$  is analytic or not. If so, find the derivative. 5

(b) Obtain the Laurent's expansion for the function  $f(z) = \frac{e^{2z}}{(z-1)^3}$  about  $z = 1$ . 5

(c) Find the inverse Laplace transform of — 5

$$\frac{S e^{-2S}}{S^2 - 6S + 25}.$$

(d) If  $A = \begin{bmatrix} 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ \sqrt{2}/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}$  find  $A^{-1}$ . 5

2. (a) Evaluate  $\int_c (z^2 + 3z) dz$  along the circle  $|z| = 2$  from  $(2, 0)$  to  $(0, 2)$ . 6

(b) Evaluate  $\int_0^{\infty} \frac{t^2 \sin 3t}{e^{2t}} dt$ . 6

(c) Determine the value of  $\lambda$  for which the following system of equations possesses a non-trivial solution and obtain these solutions for each value of  $\lambda$ . 8

$$\begin{aligned} 3x_1 + x_2 - \lambda x_3 &= 0 \\ 4x_1 - 2x_2 - 3x_3 &= 0 \\ 2\lambda x_1 + 4x_2 + \lambda x_3 &= 0. \end{aligned}$$

3. (a) Show that  $L\{\operatorname{erf} \sqrt{t}\} = \frac{1}{S\sqrt{S+1}}$  hence deduce  $L\{t \cdot \operatorname{erf}(2\sqrt{t})\}$ . 6

(b) Reduce to normal form and find the rank of — 6

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 4 & 6 & 8 & 10 \\ 15 & 27 & 39 & 51 \\ 6 & 12 & 18 & 24 \end{bmatrix}.$$

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- (c) Evaluate  $\int_C \frac{z^2}{z^4 - 1} dz$  and  $\int_C \frac{dz}{z^3(z+4)}$  where  $C$  is the circle  $|z| = 2$ . 8
4. (a) Find the residues of the function  $f(z) = \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)^2}$  at their poles. 6
- (b) Show that under the transformation  $W = \frac{3-z}{z-2}$  transforms the circle with centre  $\left(\frac{5}{2}, 0\right)$  and radius  $\frac{1}{2}$  in the  $z$ -plane into imaginary axis in the  $W$ -plane. 6
- (c) Solve  $y''(t) + 9y(t) = 18t$  if  $y(0) = 1$ ,  $y\left(\frac{\pi}{2}\right) = 0$ . 8
5. (a) Find the orthogonal trajectory of the family of curves given by —  $e^x \cos y - xy = c$ . 6
- (b) Is the system of vectors  $X_1 = [2 \ 2 \ 1]^T$ ,  $X_2 = [1 \ 3 \ 1]^T$ ,  $X_3 = [1 \ 2 \ 2]^T$  linearly dependent? 6
- (c) Evaluate  $\int_0^{2\pi} \frac{\sin^2 \theta}{5 - 4 \cos \theta} d\theta$ . 8
6. (a) Obtain the bilinear transformation that maps the points  $z = 0, -i, -1$  onto  $w = i, 1, 0$ . 6
- (b) Find the Laplace Transform of the periodic function 6
- $$f(t) = \begin{cases} t & 0 < t < \pi \\ \pi - t & \pi < t < 2\pi \end{cases}$$
- (c) Prove that  $u(x, y) = x^2 - y^2$  and  $v(x, y) = \frac{-y}{x^2 + y^2}$  are both harmonic functions, but  $u + iv$  is not analytic 8
7. (a) Find the inverse Laplace Transform of  $\frac{S^2 + S}{(S^2 + 1)(S^2 + 2S + 2)}$  using convolution theorem. 6
- (b) Determine the analytic function  $f(z) = u + iv$  in terms of  $z$ , when it is given that  $3u + 2v = y^2 - x^2 + 16xy$ . 6
- (c) Find the characteristic equation of the symmetric matrix — 8
- $$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Verify Cayley Hamilton theorem for  $A$  and find  $A^{-1}$ .