ws-Con-2013-1
Con. 8275–13. **GS–6855**

(3 Hours) **[Total Marks : 100**

N.B.:-1) Question No.1 is compulsory.
2) Attempt any four questions from remaining questions.
3) Figure to the right indicates marks.

1 (a) If X is Binomially distributed with E(X)=2 and Var(X) =4/3,find the probability distribution of X. 5

(b) A discrete random variable has the probability density function given below 5

X:	-2	-1	0	1	2	3
P(X=x):	0.2	k	0.1	2k	0.1	2k

Find k, the mean and variance.

(c) Find the fourier series for $f(x) = 1-x^2$ in (-1,1) 5

(d) Obtain wave equation for vibration of string. 5

2 (a) The marks obtained by students in a college are normally distributed with mean 65 and variance 25. If 3 students are selected at random from this college what is the probability that at least one of them would have scored more than 75 marks? 6

(b) Obtain the complex form of fourier series for $f(x)=e^{ax}$ in (0,a). 6

(c) A homogeneous rod of conducting material of length l has ends kept at zero temperature and the temperature at the centre is T and falls uniformly to zero at the two ends. Find the temperature u(x,t) at any time. 8

3 (a) Find a cosine series of period 2π to represent sin x in $0 \le x \le \pi$. 6

(b) In a study designed to investigate whether certain detonators used with explosives in coal mining meet the requirement that at least 90% will ignite the explosive, when charged it is found that 174 out of 200 detonators function properly. Test the null hypothesis that P=0.90 against the alternative hypothesis that P<0.9 at 0.05 level of significance. 6

(c) Calculate the correlation coefficient from the following data. 8

X: 23, 27, 28, 29, 30, 31, 33, 35, 36, 39
Y: 18, 22, 23, 24, 25, 26, 28, 29, 30, 32

4 (a) In a certain factory turning out blades, there is a small chance 1/500 for any blade to be Defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing no defective, one defective,two defective blades in a consignment of 10,000 packets . 6

(b) The regression lines of a sample are x+6y=6, and 3x+2y=10. Find (i)sample means $\bar{x}$ and $\bar{y}$ (ii) coefficient of correlation between x and y. Also estimate y when x=12. Also verify that the sum of the coefficients of regressions is greater than 2r. 6

(c) Expand f(x)= x sin x in the interval $0 \le x \le 2\pi$. 8

Deduce that $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$

[TURN OVER

5 (a) Theory predicts that the proportion of beans in the four groups A, B, C, D should be 9 : 3 : 3 : 1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental results support the theory? 6

(b) Find fourier integral representation of 6

$f(x) = x, \quad 0 < x < a$

$\quad = 0, \quad x > a$

$f(-x) = f(x)$

(c) A tightly stretched string with fixed end points x=0 and x=l, in the shape defined by $y = kx(l - x)$ where k is a constant, is released from this position of rest. Find y(x,t), the vertical displacement if $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ 8

6 (a) Nine items of a sample had the following values 6

45, 47, 50, 52, 48, 47, 49, 53, 51

Does the mean of 9 items differ significantly from the assumed population mean 47.5?

(b) Find the fourier expansion of $f(x) = 2x - x^2$, $0 \le x \le 3$ whose period is 3. 6

(c) A rectangular metal plate with insulated surfaces is of width a and so long as compared to its breadth that it can be considered infinite in length without introducing an appreciable error. If the temperature along one short edge y=0 is given by $u(x,0) = u_0 \sin(\pi x/a)$ for $0 < x < a$ and other long edges x=0 and x=a and the short edges are kept at zero degrees temperature, find the function u(x,y) describing the steady state. 8

7 (a) Show that the functions $f_1(x) = 1$, $f_2(x) = x$ are orthogonal on (-1,1). Determine the constants a and b such that the function $f_3(x) = -1 + ax + bx^2$ is orthogonal to both f_1 and f_2 on that interval. 6

(b) Find half range cosine series for f(x)=x, 0 < x < 2. Hence find the sum $\sum_{1}^{\infty} \frac{1}{n^4}$ 6

(c) Fit a second degree parabolic curve to the following data 8

X:	1,	2,	3,	4,	5,	6,	7,	8,	9
Y:	2,	6,	7,	8,	10,	11,	11,	10,	9