Con. 5978-13. **(OLD COURSE)** **LJ - 10062**

(3 Hours) **[Total Marks : 100**

N. B : (1) Question No. **1** is **compulsory**.
(2) Attempt any **four** questions from the remaining **six** questions.
(3) **Figures** to the **right** indicate marks.

1. (a) Prove that the fourth power of $(1 + 7i)(2 - i)^{-2}$ is a negative real number. **3**

(b) If $Y = \frac{x}{(x+1)^4}$, find Y_n. **3**

(c) Show that $\left[\bar{d} \times \left(\bar{a} \times \bar{b}\right)\right] \cdot \left(\bar{a} \times \bar{c}\right) = \left[\bar{a}\,\bar{b}\,\bar{c}\right]\left(\bar{a} \cdot \bar{d}\right)$ **3**

(d) If $x = (1 - y)(1 - 2y)$, prove that $Y = 1 + x - 2x^2 + \ldots\ldots$ **3**

(e) If $x = \left(1 - 2xy + y^2\right)^{-1/2}$, prove that $x\,u_x - Y\,u_y = Y^2 u^3$. **4**

(f) Find the maximum value of xy subject to the condition $x + y = 16$ using Lagrangeis method of undetermined multiplier. **4**

2. (a) Show that : $\tan 7\Theta = \dfrac{7\tan\Theta - 35\tan^3\Theta + 21\tan^5\Theta - \tan^7\Theta}{1 - 21\tan^2\Theta + 35\tan^4\Theta - 7\tan^6\Theta}$. **6**

(b) Separate into real and imaginary parts $\log_{(1-i)}(1+i)$ **6**

(c) If Z is homogeneous function of two variables x and y of degree 'n' then prove that : **8**

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + Y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

Hence find the value of $x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + Y^2 \frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$

at $x = 1$, $y = 2$.

Where $Z = \dfrac{x^3 + y^3}{y\sqrt{x}} + \dfrac{1}{x^7}\sin^{-1}\left(\dfrac{x^2 + y^2}{x^2 + 2xy}\right)$.

3. (a) Find the value of 'C' in the conclusion of Lagrang's Mean value theorem for the function x^3 on [2, 3]. **6**

(b) If $\bar{a}, \bar{b}$ are constants and $\bar{r} = \bar{a}\cos nt + \bar{b}\sin nt$, prove that : **6**

(i) $\bar{r} \times \frac{d\bar{r}}{dt} = n\left(\bar{a} \times \bar{b}\right)$

(ii) $\frac{d^2\bar{r}}{dt^2} + n^2\bar{r} = 0$.

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(c) (i) Prove that $\sec^2 x = 1 + x^2 + \frac{2x^4}{3} + \ldots\ldots\ldots$ **4**

(ii) Prove that $\sin(e^x - 1) = x + \frac{x^2}{2} - \frac{5}{24}x^4 + \ldots\ldots$ **4**

4. (a) Prove that $x^5 - 1 = (x-1)\left(x^2 + 2x\cos\frac{\pi}{5} + 1\right)\left(x^2 + 2x\cos\frac{3\pi}{5} + 1\right) = 0$. **6**

(b) Test the convergence of the series $2 + \frac{3}{2}x + \frac{4}{3}x^2 + \frac{5}{4}x^3 + \ldots\ldots\ldots, x > 0$. **6**

(c) If $Y = e^{a\sin^{-1}x}$, prove that : **8**

$$Y_{2m}(0) = a(a^2+2^2)(a^2+4^2)(a^2+(2m-2)^2).$$

$$Y_{2m+1}(0) = a^2(a^2+1^2)(a^2+3^2)\ldots(a^2+(2m-1)^2)$$

5. (a) If $Y = \frac{\log x}{x}$, prove that $Y_5 = \frac{5!}{x^6}\left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \log x\right]$. **6**

(b) Evaluate $\lim_{x \to a}\left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$. **6**

(c) (i) Show that if $(x\,y\,z)^b\,(x^a\,i + y^a\,j + z^a\,k)$ is an irrotation vector then either b = 0 a = -1. **4**

(ii) If $\bar{r}$ is the position vector of a point (x, y, z) and r is the modulus of $\bar{r}$ then prove that $r^n\,\bar{r}$ is an irrotational vector for any n but Solenoidal only if n = -3. **4**

6. (a) If $u = f(x^2 + 2y^2, y^2 + 2zx)$, prove that : **6**

$$(Y^2 - zx)\frac{\partial u}{\partial x} + (x^2 - Yz)\frac{\partial u}{\partial y} + (z^2 - xy)\frac{\partial u}{\partial z} = 0.$$

(b) Find the directional derivative of $\phi = x^2 y + y^2 z + z^2 x$ at (2, 2, 2) in the direction of the normal to the surface $4x^2 y + z x^2 = 2$ at the point (2, -1, 3). **6**

(c) If $x + iy = C \cdot \cot(u + iv)$ show that **8**

$$\frac{X}{\sin 2u} = \frac{-Y}{\sinh 2v} = \frac{C}{\cosh 2V - \cos 2u}.$$

7. (a) If $u = x^2 y$ and $x^2 + xy + y^2 = 1$, find $\frac{dy}{dx}$. **6**

(b) Find the maximum and minimum values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. **6**

(c) Show that $\tan^{-1} i\left(\frac{x-a}{x+a}\right) = \frac{i}{2}\log\frac{x}{a}$. **8**