FE sem - I
A. maths.

11/05/2013

ATKT

(2012-13)

D : PH (April Exam) 181

Con. 6865-13. **(REVISED COURSE)** **GS-5103**

(3 Hours) **[Total Marks : 80**

N.B. (1) Question No. **1** is **compulsory**.
(2) Attempt any **three** questions from Question Nos. **2** to Questions No. **6**
(3) **Figures** to the **right** indicate **full** marks.

1. (a) If cos hx = sec θ prove that x = log (sec θ + tan θ). 3

(b) If $u = \log(x^2 + y^2)$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ 3

(c) If $x = r\cos\theta$, $y = r\sin\theta$. Find $\frac{\partial(x, y)}{\partial(r, \theta)}$. 3

(d) Expand $\log(1 + x + x^2 + x^3)$ in powers of x upto x^8. 3

(e) Show that every square matrix can be uniquelly expressed as sum of a symmetric and a Skew-symmetric matrix. 4

(f) Find n^{th} order derivative of 4

$y = \cos x.\ \cos 2x.\ \cos 3x.$

2. (a) Solve the equation $x^6 - i = 0$. 6

(b) Reduce matrix A to normal form and find its rank where :– 6

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

(c) State and prove Euler's theorem for a homogeneous function in two variables and hence find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ where $u = \frac{\sqrt{x} + \sqrt{y}}{x + y}$ 8

3. (a) Determine the values of λ so that the equations $x + y + z = 1$; $x + 2y + 4z = \lambda$; $x + 4y + 10z = \lambda^2$ have a solution and solve them completely in each case. 6

(b) Find the stationary values of 6

$x^3 + y^3 - 3axy,\ a > 0.$

(c) Separate into real and imagianary parts $\tan^{-1}(e^{i\theta})$. 8

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4. (a) If $x = u \cos v, \quad y = u \sin v$ 6

Prove that $\dfrac{\partial(x, y)}{\partial(u, v)} \cdot \dfrac{\partial(u, v)}{\partial(x, y)} = 1.$

(b) If $\tan [\log (x + iy)] = a + ib$, prove that $\tan [\log (x^2 + y^2)] = \dfrac{2a}{(1 - a^2 - b^2)}$ where 6

$a^2 + b^2 \neq 1.$

(c) Using Gauss-Siedel iteration method, solve 8

$10x_1 + x_2 + x_3 = 12$

$2x_1 + 10x_2 + x_3 = 13$

$2x_1 + 2x_2 + 10x_3 = 14$

upto three iterations.

5. (a) Expand $\sin^7 \theta$ in a series of sines of multiples of θ. 6

(b) Evaluate $\lim\limits_{x \to 0} \dfrac{(x^x - x)}{(x - 1 - \log x)}$ 6

(c) If $y^{1/m} + y^{-1/m} = 2x$, prove that 8

$(x^2 - 1)\, y_{n+2} + (2n + 1)\, xy_{n+1} + (n^2 - m^2)\, y_n = 0.$

6. (a) Examine the following vectors for linear dependence/Independence. 6

$X_1 = (a, b, c),\ X_2 = (b, c, a),\ X_3 = (c, a, b)$ where $a + b + c \neq 0$.

(b) If $z = f(x, y), \quad x = e^u + e^{-v}, y = e^{-u} - e^v$, prove that 6

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

(c) Fit a straight line to the following data and estimate the production in the year 1957. 8

Year :	1951	1961	1971	1981	1991
Production in the Thousand tons :	10	12	08	10	13
